MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

14th Annual

AMC 10 B

American Mathematics Contest 10 B

Wednesday, February 20, 2013

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 31st annual American Invitational Mathematics Examination (AIME) on Thursday, March 14, 2013 or Wednesday, April 3, 2013. More details about the AIME and other information are on the back page of this test booklet.

1. What is $\frac{2+4+6}{1+3+5} - \frac{1+3+5}{2+4+6}$?

   (A) -1 **(B)** $\frac{5}{36}$ **(C)** $\frac{7}{12}$ **(D)** $\frac{49}{20}$ **(E)** $\frac{43}{3}$

2. Mr. Green measures his rectangular garden by walking two of the sides and finds that it is 15 steps by 20 steps. Each of Mr. Green's steps is 2 feet long. Mr. Green expects a half a pound of potatoes per square foot from his garden. How many pounds of potatoes does Mr. Green expect from his garden?

   (A) 600 **(B)** 800 **(C)** 1000 **(D)** 1200 **(E)** 1400

3. On a particular January day, the high temperature in Lincoln, Nebraska, was 16 degrees higher than the low temperature, and the average of the high and low temperatures was 3°. In degrees, what was the low temperature in Lincoln that day?

   (A) -13 **(B)** -8 **(C)** -5 **(D)** -3 **(E)** 11

4. When counting from 3 to 201, 53 is the 51^{st} number counted. When counting backwards from 201 to 3, 53 is the n^{th} number counted. What is n?

   (A) 146 **(B)** 147 **(C)** 148 **(D)** 149 **(E)** 150

5. Positive integers a and b are each less than 6. What is the smallest possible value for $2 \cdot a - a \cdot b$?

   (A) -20 **(B)** -15 **(C)** -10 **(D)** 0 **(E)** 2

6. The average age of 33 fifth-graders is 11. The average age of 55 of their parents is 33. What is the average age of all of these parents and fifth-graders?

   (A) 22 **(B)** 23.25 **(C)** 24.75 **(D)** 26.25 **(E)** 28

7. Six points are equally spaced around a circle of radius 1. Three of these points are the vertices of a triangle that is neither equilateral nor isosceles. What is the area of this triangle?

   (A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** 1 **(D)** $\sqrt{2}$ **(E)** 2

8. Ray's car averages 40 miles per gallon of gasoline, and Tom's car averages 10 miles per gallon of gasoline. Ray and Tom each drive the same number of miles. What is the cars' combined rate of miles per gallon of gasoline?

(A) 10 **(B)** 16 **(C)** 25 **(D)** 30 **(E)** 40

9. Three positive integers are each greater than 1, have a product of 27,000, and are pairwise relatively prime. What is the sum of these integers?

(A) 100 **(B)** 137 **(C)** 156 **(D)** 160 **(E)** 165

10. A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt?

(A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

11. Real numbers x and y satisfy the equation $x^2 + y^2 = 10x - 6y - 34$. What is $x + y$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

12. Let S be the set of sides and diagonals of a regular pentagon. A pair of elements of S are selected at random without replacement. What is the probability that the two chosen segments have the same length?

(A) $\frac{2}{5}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{4}{5}$

13. Jo and Blair take turns counting from 1 to one more than the last number said by the other person. Jo starts by saying "1", so Blair follows by saying "1, 2". Jo then says "1, 2, 3", and so on. What is the 53$^{\text{rd}}$ number said?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

14. Define $a\clubsuit b = a^2b - ab^2$. Which of the following describes the set of points (x, y) for which $x\clubsuit y = y\clubsuit x$?

(A) a finite set of points

(B) one line

(C) two parallel lines

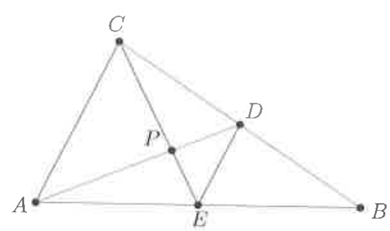
(D) two intersecting lines

(E) three lines

15. A wire is cut into two pieces, one of length a and the other of length b. The piece of length a is bent to form an equilateral triangle, and the piece of length b is bent to form a regular hexagon. The triangle and the hexagon have equal area. What is $\frac{a}{b}$?

(A) 1 **(B)** $\frac{\sqrt{6}}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $\frac{3\sqrt{2}}{2}$

16. In $\triangle ABC$, medians $\overline{AD}$ and $\overline{CE}$ intersect at P, $PE = 1.5$, $PD = 2$, and $DE = 2.5$. What is the area of $AEDC$?



(A) 13 **(B)** 13.5 **(C)** 14 **(D)** 14.5 **(E)** 15

17. Alex has 75 red tokens and 75 blue tokens. There is a booth where Alex can give two red tokens and receive in return a silver token and a blue token, and another booth where Alex can give three blue tokens and receive in return a silver token and a red token. Alex continues to exchange tokens until no more exchanges are possible. How many silver tokens will Alex have at the end?

(A) 62 **(B)** 82 **(C)** 83 **(D)** 102 **(E)** 103

18. The number 2013 has the property that its units digit is the sum of its other digits, that is $2 + 0 + 1 = 3$. How many integers less than 2013 but greater than 1000 share this property?

(A) 33 **(B)** 34 **(C)** 45 **(D)** 46 **(E)** 58

19. The real numbers c, b, a form an arithmetic sequence with $a \geq b \geq c \geq 0$. The quadratic $ax^2 + bx + c$ has exactly one root. What is this root?

(A) $-7 - 4\sqrt{3}$ **(B)** $-2 - \sqrt{3}$ **(C)** -1 **(D)** $-2 + \sqrt{3}$ **(E)** $-7 + 4\sqrt{3}$

20. The number 2013 is expressed in the form

$$2013 = \frac{a_1!a_2!\cdots a_m!}{b_1!b_2!\cdots b_n!},$$

where $a_1 \geq a_2 \geq \cdots \geq a_m$ and $b_1 \geq b_2 \geq \cdots \geq b_n$ are positive integers and $a_1 + b_1$ is as small as possible. What is $|a_1 - b_1|$?

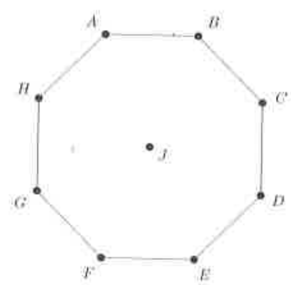
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

21. Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the seventh term of each sequence is N. What is the smallest possible value of N?

(A) 55 **(B)** 89 **(C)** 104 **(D)** 144 **(E)** 273

22. The regular octagon $ABCDEFGH$ has its center at J. Each of the vertices and the center are to be associated with one of the digits 1 through 9, with each digit used once, in such a way that the sums of the numbers on the lines AJE, BJF, CJG, and DJH are equal. In how many ways can this be done?

(A) 384 **(B)** 576 **(C)** 1152 **(D)** 1680 **(E)** 3456



23. In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{AF} \perp \overline{BF}$. The length of segment $\overline{DF}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

24. A positive integer n is *nice* if there is a positive integer m with exactly four positive divisors (including 1 and m) such that the sum of the four divisors is equal to n. How many numbers in the set $\{2010, 2011, 2012, \ldots, 2019\}$ are nice?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

25. Bernardo chooses a three-digit positive integer N and writes both its base-5 and base-6 representations on a blackboard. Later LeRoy sees the two numbers Bernardo has written. Treating the two numbers as base-10 integers, he adds them to obtain an integer S. For example, if $N = 749$, Bernardo writes the numbers 10,444 and 3,245, and LeRoy obtains the sum $S = 13{,}689$. For how many choices of N are the two rightmost digits of S, in order, the same as those of $2N$?

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25